Stability Analysis of Switched DC-DC Boost Converters for Integrated Circuits

Kevin Fronczak
Advisor: Dr. Robert Bowman
Overview

Switched DC-DC Converters

• Why does stability matter?
• How does the architecture affect instability?
• How does component variability affect stability?
• What methods are used to minimize instability?
• How can stability be measured?
Importance of Stability
Switched DC-DC Converters

• Supply oscillations can couple into signal paths
  • Sensitive circuits suffer
• Unstable supply = inaccurate supply
• Inaccurate supply = performance degradation
Importance of Stability
Display Driver Block Diagram

Adapted from Yang-Ching Lin et. al. (2012)
Importance of Stability

Boost Converter Architecture

- Feedback loop
  - Chance of oscillation at output
Causes of Instability

Operating Modes

• Operating Mode
  • CCM (Continuous Conduction Mode)
  • DCM (Discontinuous Conduction Mode)
Causes of Instability
Small-Signal Modeling

Average PWM Switch
Vorperian (1990)

Continuous Mode
Discontinuous Mode
Causes of Instability (CCM)

Bode Plot for Continuous Mode

- CCM has conjugate pole
  - LC Resonance

\[
\omega_0 = \frac{1 - D}{\sqrt{LC}} \\
Q = R(1 - D)\sqrt{\frac{C}{L}}
\]
Causes of Instability (CCM)

Input Step Response for Continuous Mode

![Graph showing input step response for continuous mode with different resistances](image)

- **R = 38 Ω**
- **R = 5.3e+02 Ω**
Causes of Instability (DCM)

Bode Plot for Discontinuous Mode

- DCM Appears 1st order at low frequencies
Causes of Instability (DCM)
Change in Gain for Discontinuous Mode

Gain vs. $I_O$ for DCM

$$G_{d0} = \frac{2V_O}{\sqrt{\frac{LFsI_o}{2V_O} \left[ \left(\frac{2V_O}{V_S} - 1\right)^2 - 1 \right]} + 1}$$
Causes of Instability (DCM)
Input Step Response for Discontinuous Mode

[Graph showing the input step response for Discontinuous Mode with two curves representing different resistances (R = 38 Ω and R = 5.3e+02 Ω).]
Parasitic Component Effects

Non-ideal Converter Schematic

- Inductor has series resistance
  - Lowers height of resonant peak (CCM)
- Capacitor has series resistance
  - Adds high-frequency zero
  - Less attenuation at frequencies > Fs
Parasitic Component Effects
Continuous Mode: Expected Behavior

\[ G_{d0} = \frac{V_O}{1 - D} \frac{R(1 - D)^2 - r_L}{R(1 - D)^2 + r_L} \]

\[ Q = \sqrt{\frac{L}{C}} \frac{\sqrt{(R + r_C)(R(1 - D)^2 + r_L)}}{Rr_C C (1 - D)^2 + Rr_L C + r_C r_L C + L} \]
Parasitic Effects (CCM)
Bode Plot for Continuous Mode

• Bode Plot

![Bode Plot for Continuous Mode](image)
Parasitic Effects (DCM)
Bode Plot for Discontinuous Mode

- Bode Plot

![Bode Plot for Discontinuous Mode](image)
Component Variation
Effect on Phase Margin

- Input Voltage, Load Current, Inductance
  - \(\frac{V_o}{V_s} = [3.3, 2, 1.4]\)
  - \(I_o = [6 \text{ mA}, 30 \text{ mA}, 54 \text{ mA}] \text{ (DCM)}; [150 \text{ mA}, 300 \text{ mA}, 450 \text{ mA}] \text{ (CCM)}\)
  - \(L = [15 \text{ uH}, 20 \text{ uH}, 25 \text{ uH}] \text{ (DCM)}; [170 \text{ uH}, 200 \text{ uH}, 230 \text{ uH}] \text{ (CCM)}\)

*Note – all values calculated for converter WITH control
Component Variation

Effect on DC-Gain

- Input Voltage, Load Current, Inductance
  - $\frac{V_o}{V_s} = [3.3, 2, 1.4]$
  - $I_o = [6 \text{ mA}, 30 \text{ mA}, 54 \text{ mA}]$ (DCM); $[150 \text{ mA}, 300 \text{ mA}, 450 \text{ mA}]$ (CCM)
  - $L = [15 \text{ uH}, 20 \text{ uH}, 25 \text{ uH}]$ (DCM); $[170 \text{ uH}, 200 \text{ uH}, 230 \text{ uH}]$ (CCM)

*Note – all values calculated for converter WITH control*
Control of Converters

Error Amplifier Requirements

• Control needs to help provide stable output
  • Requires feedback loop → source of instability
  • Needs to minimize output error (large gain)
  • Needs to minimize instability (large phase margin)
  • Needs to maximize speed (large bandwidth)

• Op Amps/OTAs
  • Op Amps
    • Voltage buffer → Slows down performance
    • Can drive low impedances
  • OTAs
    • Can’t drive resistive loads
    • Fast → does not have voltage buffer (response limited by load capacitance)
Control of Converters (CCM)  
Proportional-Integral-Derivative Architecture

- PID Required for CCM
- Two poles, two zeros
  - Minimizes conjugate pair effect

\[ \frac{V_c}{V_o} = \frac{g_m R_B}{s (R_B + R_T) (C_Z + C_C)} \left[ \frac{(s R_T C_1 + 1)}{(s R_Z C_Z + 1)} \right] \left[ \frac{(s R_Z C_Z C_C + 1)}{(s R_T || R_B) C_1 + 1} \right] \]
Control of Converters (CCM)

Bode Plot and Step Response

- Difficult to achieve all three requirements
  - Gain, Phase Margin, Bandwidth
Control of Converters (DCM)

Lag Controller Architecture

- Lag Controller suitable for DCM
- Zero to cancel converter Pole
- Pole to attenuate switching noise

\[
\frac{V_c}{V_o} = K \frac{s R_Z C_Z + 1}{s^2 R_O R_Z C_C C_Z + s R_O C_Z + 1}
\]

\[
K = g_m R_O \frac{R_B}{R_T + R_B}
\]
Control of Converters (DCM)

Bode Plot and Step Response

- Easier control compared to CCM
- More stable with fewer components
Measuring Stability

Stability Measurement Requirements and Possibilities

• Cannot “break the loop”
  • High loop gain
• Observe step response
• Superimpose voltage (Middlebrook’s Method)
• Cross-correlation
Measuring Stability
Load Step Response

- Converter reaches steady state
  - Step the load current
- Composite Response of System
  - No pole/zero information

Overdamped Response

Underdamped Response
Measuring Stability
Middlebrook’s Method (1975)

Conventional Approach
- Voltage Injection

Middlebrook’s Method
- Superposition
Measuring Stability

Implementation of Middlebrook’s Method

- Can measure with Network Analyzer
- Bode plot can be compared to simulations
- Gives information on overall stability
  - Pole/Zero migration can be directly observed
Measuring Stability

Cross Correlation with White Noise

• Inject white noise in control signal path
  • White noise has autocorrelation of delta function
    • Yields impulse response
    • FFT yield frequency response

\[
\begin{align*}
R_{xy}[m] &= \sum_{n=1}^{\infty} h[n] R_{xx}[m - n] + R_{xv}[m] \\
R_{xx}[m] &= \delta[m] \quad \rightarrow (3) R_{xy}[m] &= h[m]
\end{align*}
\]
Measuring Stability

“White Noise” Circuit Implementation

- PRBS (Pseudo-Random Binary Sequence)
  - Periodic noise
    - Cross-correlation can only happen within one period
    - Period can be made larger by adding more bits to sequence
    - PRBS amplitude must be small percentage of control signal
    - Can be superimposed over $V_{\text{ref}}$

$$T_{PRBS} = \frac{2^n - 1}{f_k}$$
Measuring Stability

PRBS Autocorrelation

- Single period PRBS (a) has small autocorrelation value
  - Ideally infinite
- Multi-period PRBS (b) has larger value
  - Can sample the converter response multiple times
    - Average of results gives more accurate frequency response
Measuring Stability

Frequency Response Method Comparison

• Frequency Response gives most information on circuit
  • Can directly compare bode plot to simulations
  • Can see effect of resonance (for CCM)
  • Can see effect of filter capacitor ESR zero

• Methods
  • Middlebrook’s Method
    • + Simple
    • - Requires Manual Capture
  • Cross-correlation (PRBS)
    • + Allows for Built-In Self-Test (BIST)
    • - Adds complexity
Summary

• DCM = Easier to Control
• Controller Design = Simple for DCM, Complex for CCM
• Frequency Response = Vital in determining stability
  • Two techniques: Middlebrook, Cross Correlation

Future Work

• Controller design via optimization algorithms
  • Genetic Algorithms, Particle Swarm Optimization
• More in-depth exploration of PRBS
  • On-chip Supply Testing
Thank You

• Comments/Questions?